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Effects of bulk viscosity in non-linear bubble dynamics

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Abstract
The non-linear bubble dynamics equations in a compressible liquid have been modified by considering the effects of compressibility of both the liquid and the gas at the bubble interface. A new bubble boundary equation has been derived, which includes a new term resulting from the liquid bulk viscosity effect. The influence of this term has been numerically investigated by considering the effects of water vapour and chemical reactions inside the bubble. The results clearly indicate that the new term has an important damping role at the collapse, so that its consideration dramatically decreases the amplitude of the bubble rebounds after the collapse. This damping feature is more remarkable for higher driving pressures.

When a small isolated gas bubble, immersed in a liquid, experiences a high amplitude spherical sound field, it grows and contracts non-linearly. The description of the dynamics of such non-linear motion is an old and challenging problem. The radial dynamics of the bubble in an incompressible liquid is described by the well-known incompressible Rayleigh–Plesset equation [1, 2]. The extension of this equation to the bubble motion in a compressible liquid has been studied by many previous authors [3, 4]. The most complete existing description was presented by Prosperetti and Lezzi [5]. They used a singular-perturbation method of the bubble-wall Mach number and derived a one-parameter family of equations describing the bubble motion in the first-order approximation of the compressibility. This family of equations is written as

$$\left(1-(\eta+1)\frac{\dot{R}}{C}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{1}{3}(3\eta+1)\frac{\dot{R}}{C}\right)\dot{R}^2 = \frac{R}{\rho C}\frac{\mathrm{d}}{\mathrm{d}t}(P_\mathrm{l}-P_\mathrm{a})+\left(1+(1-\eta)\frac{\dot{R}}{C}\right)\left(\frac{P_\mathrm{l}-P_\mathrm{a}-P_0}{\rho}\right), \quad (1)$$

where R, C, P_0, P_a, and ρ are the bubble radius, liquid sound speed, ambient pressure, driving pressure, and density of the liquid, respectively. Also, η is an arbitrary parameter. Equation (1) must be supplemented by a boundary condition equation at the bubble interface to relate the liquid pressure, P_l, to the gas pressure inside the bubble. Like all previous authors, Prosperetti and Lezzi [5] used the following incompressible equation for this purpose:

$$P_l = P_g - 4\mu\frac{\dot{R}}{R} - \frac{2\sigma}{R}, \tag{2}$$

where P_g, μ, and σ are the gas pressure at the bubble interface, liquid viscosity coefficient, and surface tension, respectively. Most of the previously obtained equations belong to this single parameter family of equations, corresponding to different values of η. Moreover, $\eta = 0$ yields results in closest agreement with the numerical simulation of full partial differential equations [5].

In all previous works [1–5], an important approximation has been used in the derivation of the bubble dynamics equations. That is the incompressibility assumption of the liquid motion at the bubble interface, which has been used in the derivation of equation (2). Note that all of the effects of the liquid compressibility in all previous papers have resulted from the liquid motion around the bubble, but not from the bubble boundary condition equation. In fact, all previous authors on the one hand took into account the compressibility of the liquid motion around the bubble, but on the other hand neglected its consideration at the bubble interface.

In this work, we have modified the bubble dynamics equations by considering the effects of the compressibility at the bubble interface. We have derived a new bubble boundary equation including new terms resulting from the effects of bulk viscosity of the liquid and the gas.

To derive the compressible bubble boundary equation, we apply the spherical symmetric assumption for the bubble motion. The continuity equation in this case is [6]

$$\frac{1}{\rho}\left[\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r}\right] = -\frac{\partial u}{\partial r} - \frac{2u}{r} = -\Delta, \tag{3}$$

where ρ, u, and Δ are the density, velocity, and divergence of velocity, respectively. Note that, under the spherical symmetric condition, all dependent variables are only functions of r and t, the only existing independent variables in this case. Also, the radial component of the stress tensor can be written as [6, 7]

$$T_{rr} = -p + \left(\mu_b - \frac{2\mu}{3}\right)\Delta + 2\mu\left(\frac{\partial u}{\partial r}\right), \tag{4}$$

where p and μ_b are the pressure and coefficient of bulk viscosity. The bulk viscosity μ_b, which is usually of the same order of magnitude as the ordinary viscosity μ, arises from the proportionality of the normal stress tensor to the velocity divergence. Therefore, its importance appears in processes which are accompanied by a change in density of fluid. In fact, the bulk viscosity is important in bulk compression or expansion of a fluid and appears only when the fluid flow is compressible.

Inserting $\partial u/\partial r$ from equation (3), into (4) yields

$$T_{rr} = -p + \left(\mu_b + \frac{4\mu}{3}\right)\Delta - 4\frac{\mu u}{r}. \tag{5}$$

The velocity divergence, Δ, can be written as

$$\Delta = -\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{1}{\rho c^2}\frac{\mathrm{d}p}{\mathrm{d}t}, \tag{6}$$

where the sound speed, c, is defined as $c^2 = \mathrm{d}p/\mathrm{d}\rho$. The boundary continuity requirement at the bubble interface is

$$T_{rr}(\text{liquid})|_R = T_{rr}(\text{gas})|_R + 2\frac{\sigma}{R}. \tag{7}$$

Applying equation (5) for the gas and the liquid parts of equation (7) leads to

$$P_{\mathrm{l}} + 4\frac{\mu \dot{R}}{R} - \left(\mu_{\mathrm{b}} + \frac{4\mu}{3}\right)\Delta_{\mathrm{l}} = P_{\mathrm{g}} + 4\frac{\mu_{\mathrm{g}} \dot{R}}{R} - \left(\mu_{\mathrm{bg}} + \frac{4\mu_{\mathrm{g}}}{3}\right)\Delta_{\mathrm{g}} - 2\frac{\sigma}{R}, \tag{8}$$

where μ_{g} and μ_{bg} are the viscosity and the bulk viscosity coefficients of the gas at the bubble interface, respectively. Also, Δ_{l} and Δ_{g} are the divergences of velocity of the liquid and the gas, respectively. Substituting Δ_{l} and Δ_{g} from equation (6) into (8) yields

$$P_{\mathrm{l}} + 4\frac{\mu \dot{R}}{R} + \left(\frac{\mu_{\mathrm{b}}}{\rho C^2} + \frac{4\mu}{3\rho C^2}\right)\frac{\mathrm{d}P_{\mathrm{l}}}{\mathrm{d}t} = P_{\mathrm{g}} + 4\frac{\mu_{\mathrm{g}} \dot{R}}{R} + \left(\frac{\mu_{\mathrm{bg}}}{\rho_{\mathrm{g}}} + \frac{4\mu_{\mathrm{g}}}{3\rho_{\mathrm{g}}}\right)\frac{\mathrm{d}\rho_{\mathrm{g}}}{\mathrm{d}t} - 2\frac{\sigma}{R}, \tag{9}$$

where ρ_{g} is the gas density at the bubble interface. Equation (9) represents the bubble boundary equation containing all the effects of the compressibility and the viscosity of both the liquid and the gas. Comparison of equations (2) and (9) indicates the existence of three new terms in equation (9), due to the liquid and the gas compressibility and viscosity effects.

Here, we concentrate on the effects of the new term arising from the liquid compressibility. Thus, the gas viscous terms in equation (9) are neglected as in previous works [1–5]. This is based on the assumption that $\partial u/\partial r$ is of the same order for both the gas and the liquid at the bubble interface. With this assumption, both Δ_{g} and Δ_{l} are of the same order at the bubble boundary (consider the definition of Δ in equation (3)). Recall that the gas velocity u_{g} is equal to the liquid velocity u at the bubble wall (kinematic boundary condition). Since the gas viscosity μ_{g} is usually three orders of magnitude smaller than the liquid viscosity μ [22], the gas viscous terms in equations (8) and (9) can be neglected in comparison with the liquid viscous terms. Under these circumstances, equation (9) becomes

$$P_{\mathrm{l}} + \left(\frac{\mu_{\mathrm{b}}}{\rho C^2} + \frac{4\mu}{3\rho C^2}\right)\frac{\mathrm{d}P_{\mathrm{l}}}{\mathrm{d}t} = P_{\mathrm{g}} - 4\frac{\mu \dot{R}}{R} - 2\frac{\sigma}{R}. \tag{10}$$

It should be mentioned that, although the effects of compressibility consideration in equation (1) are in the first-order approximation, these effects have been introduced completely in equation (10).

To close the mathematical analysis, the gas pressure evolution at the bubble interface, P_{g}, must be specified. In the most complete approach, it can be determined from the simultaneous solution of the conservation equations for the bubble interior and the bubble radius equations [8–13]. Also, heat conduction and mass exchange between the bubble and the surrounding liquid affect the bubble evolution. In addition, chemical reactions occurring in high-temperature conditions at the end of the collapse change the bubble content [14, 15]. All these complexities have been considered in a complete gas dynamics simulation by Storey and Szeri [16].

On the other hand, strong spatial inhomogeneities inside the bubble are not particularly revealed, except at the end of an intense collapse [11, 12]. Therefore, the uniformity assumption for the bubble interior seems to be useful and provides many features of the bubble motion [17, 18]. Using this assumption, Lohse and his co-workers recently presented an ODE model [19–21], in which the effects of heat transfer at the bubble interface, phase change of water vapour, chemical reactions, and diffusion of reaction products have been considered. This model accurately describes various experimental phase diagrams [20] and provides a good agreement with the complete direct simulation of Storey and Szeri [16, 19].

Here, for describing the bubble interior evolution, we have used the Lohse's group model (the same as has been presented in [21]). We do not repeat this model here and for more details we refer to [20, 21]. The calculations were carried out under the framework of equation (1) ($\eta = 0$) for both the new compressible (equation (10)) and the old incompressible (equation (2)) boundary conditions. We describe an argon bubble in water at room temperature,

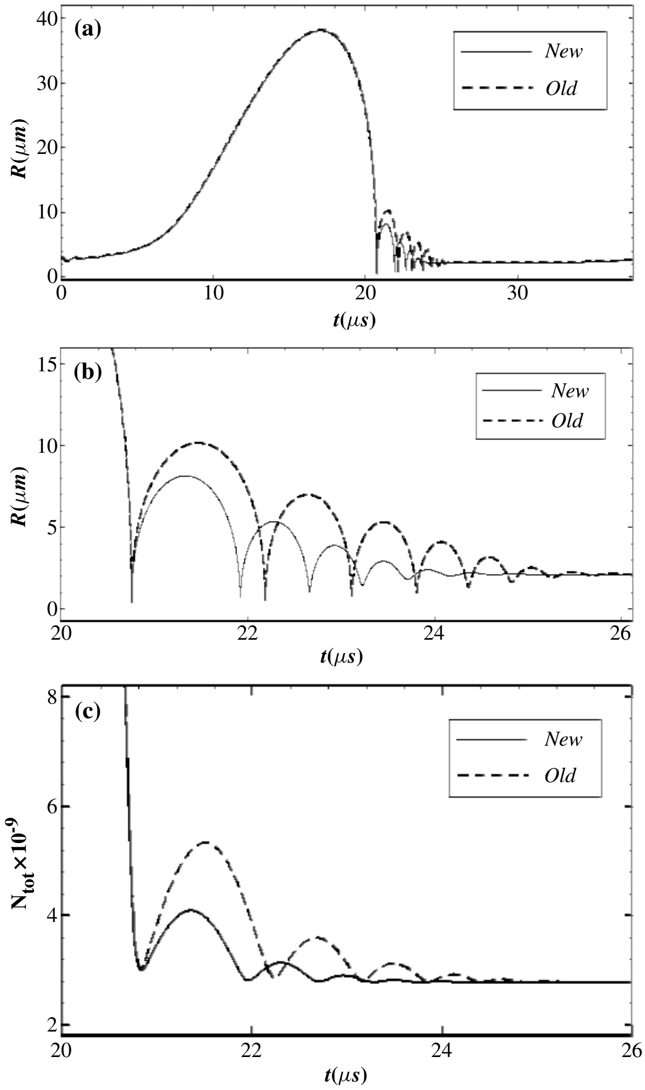

Figure 1. (a) Time variations of the bubble radius during one period of the pressure field according to the new compressible (solid) and the old incompressible (dashed) boundary conditions. (b) Details of the bubble radius evolution at the end of the collapse and during the bubble rebounds for the two boundary conditions. (c) Time variation of the total number of particle species inside the bubble for the two boundary equations during the bubble rebounds. The equilibrium radius and the driving pressure are $R_0 = 3.0\ \mu$m and $P_a = 1.35$ atm.

$T_0 = 293.0$ K, and atmospheric pressure, $P_0 = 1.0$ atm, under the conditions of single bubble sonoluminescence [17, 18]. The driving pressure was $P_a(t) = P_a \sin(\omega t)$, where $\omega = 2\pi \times 26.5$ kHz. The other constants and parameters were set accordingly [22]: $\rho = 998.0$ kg m^{-3}, $C = 1483.0$ m s^{-1}, $\mu = 1.01 \times 10^{-3}$ kg ms^{-1}, $\sigma = 0.0707$ kg s^{-2}. The bulk viscosity of water at room temperature was set as $\mu_b = 4.1 \times 10^{-3}$ kg ms^{-1} [23]. The constants and parameters of the gas evolution model were set the same as has been presented in [21].

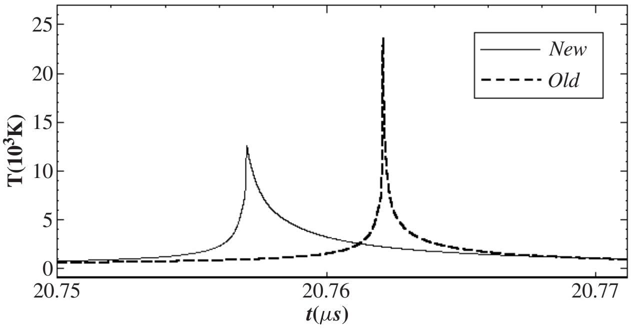

Figure 2. Time variations of the gas temperature when the bubble reaches its minimum radius according to compressible (solid) and incompressible (dashed) boundary conditions. The parameters and constants are the same as in figure 1.

Figures 1 and 2 illustrate the variations of the bubble characteristics (radius, total number of particle species, and temperature), for the two boundary condition cases. It is observed that the addition of the new viscous term in equation (10) considerably changes the bubble evolution after the collapse (figures 1(b) and (c)). The bubble motion is remarkably compressible during the collapse. Therefore, the new viscous term, which has arisen from the liquid compressibility, is important in this time interval. This term exhibits a damping role, and its consideration dramatically reduces the amplitude of the bubble rebounds. Also, the period of the rebounds considerably decreases with the addition of the new term. Details of our calculations at the end of the collapse show that the minimum radius for the new case is about 10% greater than that of the old one. The difference between the two cases also appears on the time variations of the total number of particle species inside the bubble (Ar and H_2O plus reaction products), which has been illustrated in figure 1(c). Note that the difference gradually disappears as the bubble rebounds are weakened.

In figure 2, details of the gas temperature evolution around the minimum radius time have been demonstrated. The damping feature of the new term is clearly observed by a considerable decrease in the peak temperature (about 50%) and a distinguishable increase in the width of the temperature pulse. Also, the time of the peak temperature about 5 ns changes with the addition of the new term.

Figure 3 represents the effects of variations of the driving pressure amplitude, P_a, on the bubble characteristics at the end of the collapse (peak temperature, mole fraction of H_2O and reaction products, and minimum radius), for the two boundary condition cases. The ambient radius was fixed ($R_0 = 5.0\ \mu$m). Figure 3(a) shows that the peak temperature in the both cases grows as the driving pressure is increased. However, the rate of increase for the new case is considerably smaller than for the old one. This causes the difference between the two boundary conditions to become more remarkable for higher driving pressures (about 50% for $P_a = 1.5$ atm).

The value of bubble temperature at the end of collapse is high enough to destroy chemical bonds of water vapour molecules inside the bubble. The products of dissociation of water vapour molecules are mainly H_2, OH, H, O, and O_2. The chemical reactions between the particle species existing inside the bubble affect the bubble content and its temperature at the end of collapse [14–16, 18–21]. In this work, we have considered effects of the reactions No 1–8 of [15, 20], which are the most important reactions at the collapse time [21]. The dependence of the mole fraction of H_2O plus reaction products, which is defined as $(N_{tot} - N_{Ar})/N_{tot}$, on

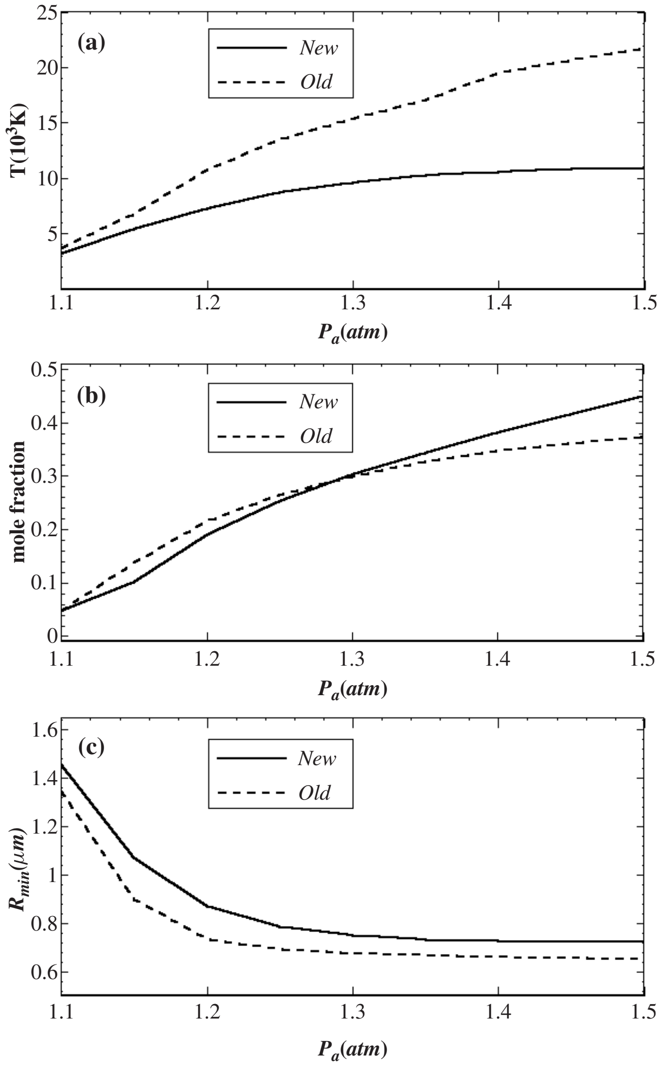

Figure 3. The bubble characteristics at the time of collapse as a function of driving pressure amplitude for the compressible (solid) and incompressible (dashed) boundary conditions: peak temperature (a), mole fraction of H_2O and reaction products (b), and minimum radius (c). The equilibrium radius was fixed ($R_0 = 5.0\ \mu$m) and other constants are the same as in figures 1 and 2.

the driving pressure for the two boundary condition cases has been illustrated in figure 3(b). It shows that the mole fraction of H_2O plus reaction products is similar for the two cases in low amplitudes. The difference gradually appears as the driving pressure is increased. It is seen that, for the higher driving pressures, the effect of H_2O and reaction products is more important in the new case relative to the old one.

Figure 3(c) shows the variations of the bubble minimum radius as a function of P_a, for the two cases. The trend of variations is similar for the two cases, but the minimum radius for the new equation is greater than that of the old equation because of the reduction of the collapse intensity in the new case.

A major deficiency of the old bubble dynamics equations is that for strongly driven bubbles, such as sonoluminescence bubbles, large amplitude rebounds are produced after the collapse, so that they often last until the next acoustic cycle of the periodic driving pressure. This is in contrast with the experimental results, which show rapidly damped rebounds [24]. By introducing a damping term arisen from the gas compressibility, Moss *et al* [24] provided a typical solution for this problem. The influence of the suggested term by Moss and co-workers is very similar to the damping effect of the new term in this work (compare figure 1(b) with figures 3 and 4 of [24]). It seems that the damping feature of the bulk viscosity is a better way for solving the problem. The reason for this is that equation (10) has been derived directly from the basic equations of fluid mechanics, whereas equation (3.2) of [24] has been derived by an approximate method.

The results of this work clearly show that the damping effects of the liquid bulk viscosity are important during the bubble evolution at the collapse time. This point strongly suggests that the new effects should be considered for prediction of the quantities related to the collapse, such as the value of light emission by a single sonoluminescent bubble as well as the bubble stability limits.

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